

# Applications of Magnetic Sensors for Low Cost Compass Systems

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*Abstract*—A method for heading determination is described here that will include the effects of pitch and roll as well as the magnetic properties of the vehicle. Using solid-state magnetic sensors and a tilt sensor, a low-cost compass system can be realized. Commercial airlines today use attitude and heading reference systems that cost tens of thousands of dollars. For general aviation, or small private aircraft, this is too costly for most pilots' budget. The compass system described here will provide heading, pitch and roll outputs accurate to one degree, or better. The shortfall of this low-cost approach is that the compass outputs are affected by acceleration and turns. A solution to this problem is presented at the end of this paper.

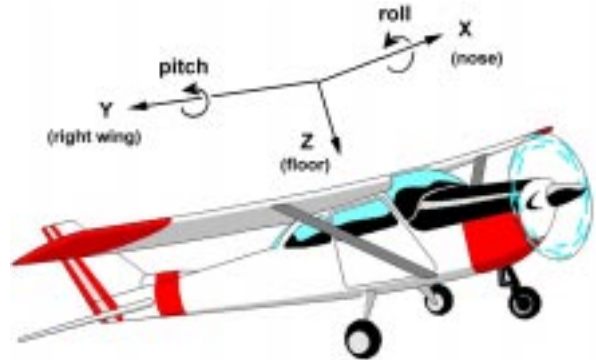


Figure 1—Coordinate direction (X,Y,Z) and attitude orientation (roll,pitch) on an aircraft.

## BACKGROUND

The Earth's magnetic field intensity is about 0.5 to 0.6 gauss and has a component parallel to the Earth's surface that always point toward magnetic north. This field can be approximated with a dipole model—the field points down toward north in the Northern Hemisphere, is horizontal and pointing north at the equator, and point up toward north in the Southern Hemisphere. In all cases, the horizontal direction of the Earth's field is always pointing toward magnetic north and is used to determine compass direction.

Aircraft convention defines the attitude parameters in terms of three angles: heading, pitch and roll (see Figure 1). These angles are referenced to the local horizontal plane. That is, the plane perpendicular to the earth's gravitational vector. Heading is defined as the angle in the local horizontal plane measured clockwise from a true North (earth's polar axis) direction. Pitch is defined as the angle between the aircraft's longitudinal axis and the local horizontal plane (positive for nose up). Roll is defined as the angle about the longitudinal axis between the local horizontal plane and the actual flight orientation (positive for right wing down).

The local horizontal plane is defined as the plane normal to the earth's gravity vector (see Figure 2). If a compass was sitting in the local horizontal plane, then the roll and pitch angles would be zero and the heading would be calculated as:

$$\text{Heading} = \text{arcTan} (Y_h/X_h) \quad (1)$$

where  $X_h$  and  $Y_h$  represent the earth's horizontal magnetic field components. As the aircraft is rotated, the heading would sweep  $0^\circ$  to  $360^\circ$  referenced to magnetic north. If the compass were now tilted, the tilt angles (roll and pitch) and all three magnetic field components (X,Y,Z) must be used in order to calculate heading [1].

## TILT DETERMINATION

One method to determine the roll and pitch angles is to use a tilt sensor that senses the direction of gravity. Common tilt measuring devices include accelerometers, electrolytic (fluid) based tilt sensors, and gimballed mechanical structures. Another method to determine the local horizontal plane is to use a gyroscope to maintain a known inertial reference orientation at all times.

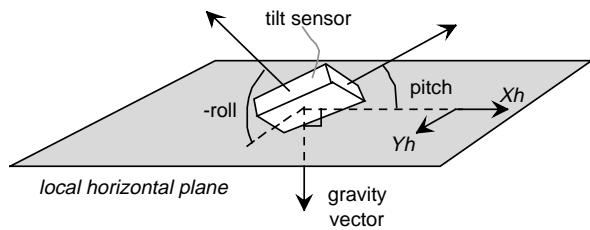


Figure 2—Tilt sensor angles are referenced to the local horizontal plane defined by gravity.

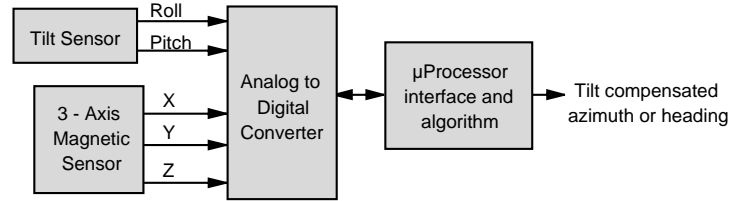


Figure 3—Compass system block diagram.

Gyroscopes (gyros) are instruments used to measure precise angular motion. Several techniques are used to achieve this such as spinning wheels, vibrating structures, and ring lasers. The output signal is proportional to the angular rate of turn. A key consideration when using gyros is the output drift with time. With periodic correction, the drift can be compensated for and provide very high levels of accuracy for roll, pitch, and heading. Gyros are standard in navigation instrument on commercial aircraft and will operate well under accelerating conditions. When compared to tilt sensors, though, gyros tend to be bulky and expensive and will not be considered here.

Tilt sensors come in many types and sizes. The gimballed tilt device usually has two rings mounted at right angles to each other much like a dual pendulum. A magnetic sensor, or compass, inside of the gimballed structure will remain suspended in the local horizontal plane for various roll and pitch angles. The mechanical structure of the gimbal makes it susceptible to shock and vibration and can often take seconds for it to become stable after movement. Gimballed compasses only require two axes of magnetic sensing since the roll and pitch angles are never present in a steady-state condition. However, since the magnetic sensors change orientation with the compass platform, these compasses cannot compensate for the ferrous effects of its surroundings.

Low cost tilt sensors like the two-axis electrolytic and dual axis accelerometer measure the roll and pitch angle directly. Liquid filled electrolytic tilt sensors, resembling a glass “thimble”, use electrodes to monitor the fluid movement as the sensor changes angles. Solid state accelerometer tilt sensors measure the Earth’s gravitational field by means of an electromechanical circuit [2]. These sensors are similar in that they have two single axis components that measure the angle deviations from the local horizontal plane. Signal conditioning circuits are used to create an output signal proportional to the angle of tilt. These sensors are considered strapdown devices since they have no moving or pendulous parts and are desirable for vehicle applications [3].

### COMPASS SYSTEM

If a strapdown compass is required to output heading for any orientation then, as a minimum, a compass system must have a three-axis magnetic sensor and a two-axis tilt (see Figure 3). The heading calculation relies on all three magnetic components (X,Y,Z) so the compass orientation can be mathematically rotated to the horizontal plane. Then, the Xh and Yh components can be calculated to determine the heading value from equation (1).

In Figure 2, a compass is shown with roll ( $\theta$ ) and pitch ( $\phi$ ) tilt angles referenced to the right and forward level directions. The X, Y, and Z magnetic readings can be transformed to the horizontal plane (Xh, Yh) by applying the rotation equations shown in equation (2). If these equations are not used, then appreciable errors will result in the heading calculations as shown in Figure 4.

$$\begin{aligned} X_h &= X \cdot \cos(\phi) + Y \cdot \sin(\theta) \cdot \sin(\phi) - Z \cdot \cos(\theta) \cdot \sin(\phi) \\ Y_h &= Y \cdot \cos(\theta) + Z \cdot \sin(\theta) \end{aligned} \quad (2)$$

Once the magnetic components are found in the horizontal plane, equation (1) can be used to determine heading. To minimize processing time, a sine and cosine lookup table can be stored in program memory. To account for the arcTan limits, the heading calculations must account for the sign of the Xh and Yh readings as shown in (3).

$$\begin{aligned} \text{Heading for } (X_h < 0) &= 180 - \text{arcTan}(Y_h/X_h) \\ \text{for } (X_h > 0, Y_h < 0) &= - \text{arcTan}(Y_h/X_h) \\ \text{for } (X_h > 0, Y_h > 0) &= 360 - \text{arcTan}(Y_h/X_h) \\ \text{for } (X_h = 0, Y_h < 0) &= 90 \\ \text{for } (X_h = 0, Y_h > 0) &= 270 \end{aligned} \quad (3)$$

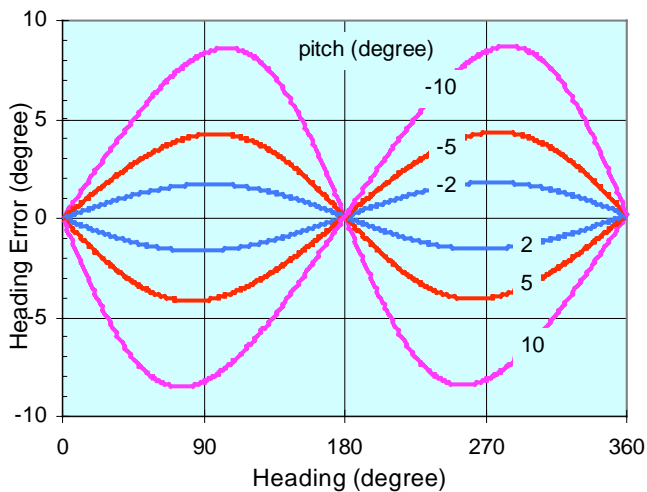


Figure 4—Heading errors due to pitch without tilt compensation (Dip Angle =40°).

### COMPASS ERROR ANALYSIS

If a compass system has a requirement of better than one degree of accuracy, then it is important to break down the error contributed by the tilt sensor and the magnetic sensor and determine what level of signal processing is required. Specifically, heading accuracy is affected by:

- ✓ A/D converter resolution
- ✓ Magnetic sensor errors
- ✓ Temperature effects
- ✓ Nearby ferrous materials
- ✓ Compass tilt errors
- ✓ Variation of the earth's field

*A/D Converter Resolution*—To achieve a one-degree accurate compass requires a magnetic sensor that can reliably resolve angular changes to 0.1°. The sensors must also exhibit low hysteresis (<0.08%FS), a high degree of linearity (<0.05%FS) and be repeatable. The magnetic fields in the X and Y horizontal plane will typically be in the 200 mgauss range—more at the equator, less near the poles.

Using the standard heading relationship of equation (1), the required A/D converter resolution for the magnetic sensors can be estimated. If the magnetometer error, or uncertainty, is allowed to be 0.1° then:

$$\begin{aligned} \text{if: } & \text{Error} = 0.1^\circ = \text{arcTan}(Y_h/X_h) \\ \text{then: } & Y_h/X_h = 1/573 \end{aligned} \quad (4)$$

This implies that a ratio change of 1 part in 573 will result in a 0.1° difference. If X and Y were read with a nine-bit A/D converter there would be only a 1:512 bit resolution. This means that a 9+ bit A/D converter is needed to meet the 0.1° error budget for an (X,Y) magnetic field change of 200 mgauss. Since the (X,Y) magnetic fields measure ±200 mgauss for a complete heading sweep, the A/D converter range should be doubled, to 10+ bits. To allow for hard iron correction and larger horizontal fields—like at the equator—this range should be quadrupled to ±800 mgauss. Now the A/D converter resolution should be 12+ bits, or 12.163 bits to be more exact.

A 12 bit A/D converter can be used to provide a 0.1° resolution in a 200 mgauss horizontal field. This implies that the sensor must be able to resolve a 0.39 mgauss field over a span of ±800 mgauss (1.6 gauss/4096 counts).

*Magnetic Sensor Errors*—Solid state magneto-resistive (MR) sensors available today can reliably resolve <0.07 mgauss fields [4-7]. This is more than a five times margin over the 0.39 mgauss field required to achieve 0.1° resolution.

Other magnetic sensor specifications should support field measurement certainty better than 0.5° to maintain an overall 1° heading accuracy. These include the sensor noise, linearity, hysteresis, and repeatability errors.

Any gain and offset errors of the magnetic sensor will be compensated for during the hard iron calibration (discussed later) and will not be considered in the error budget.

MR sensors can provide a total error of less than 0.5 mgauss, which corresponds to a 0.14° heading error as shown in Table 1.

Parameter	Spec Limit (1)	Field Error	Heading Error
Noise (BW=10Hz)	85 ugauss	85 ugauss	<0.01°
Linearity	0.05 %FS	0.2 mgauss	0.06°
Hysteresis	0.08 %FS	0.32 mgauss	0.09°
Repeatability	0.08 %FS	0.32 mgauss	0.09°
	<b>Total rms error</b>	<b>0.49 mgauss</b>	<b>0.14°</b>

(1) Typical specs for HMC1021/22 MR sensors; FS=400 mgauss

Table 1—Error budget for an MR magnetic sensor

*Temperature Effects*—The temperature coefficient (tempco) of the sensor will also affect the heading accuracy. There are two characteristics of temperature to consider—the offset drift with temperature and the sensitivity tempco. The sensitivity tempco will appear as a change in output gain of the sensor over temperature (Figure 5). MR sensors generally have sensitivity tempcos that are well correlated, or matched—especially sensors with two (X,Y) axes in the same package. The matching tempcos imply that the output change over temperature of the X axis will track the change in output of the Y axis. This effect will cancel itself since it is the *ratio* of Y over X that is used in the heading calculation [Azimuth = arcTan(Y/X)]. For example, as the temperature changes the Y reading by 12%, it also changes the X reading by 12% and the net change is canceled. The only consideration is then the dynamic input range of the A/D converter.

The magnetic sensor offset drift with temperature is not correlated and may in fact drift in opposite directions. This will have a direct affect on the heading and can cause appreciable errors. There are many ways to compensate for temperature offset drifts using digital and analog circuit techniques. A simple method to compensate for temperature offset drifts in MR sensors is to use a switching technique referred to as set/reset switching. This technique cancels the sensor temperature offset drift, and the dc offset voltage as well as the amplifier offset voltage and its temperature drift.

The transfer curves for a MR magnetic sensor after it has been set, and then reset, is shown in Figure 6. The set/reset modes are achieved by using an ac coupled driver to generate a bi-directional current

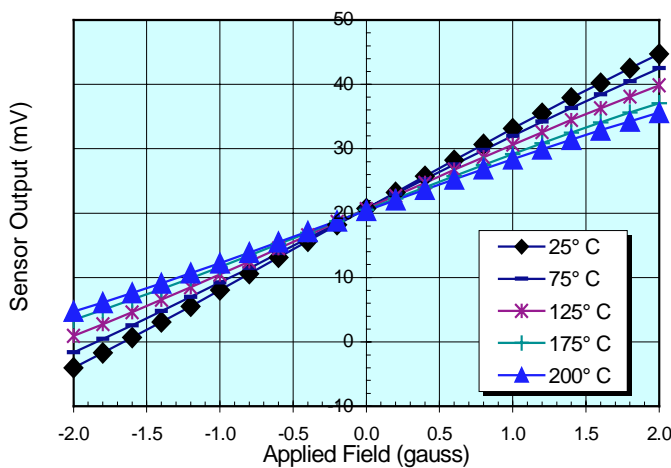


Figure 5—Magnetic sensor output temperature variation has a pivot point at zero applied field.

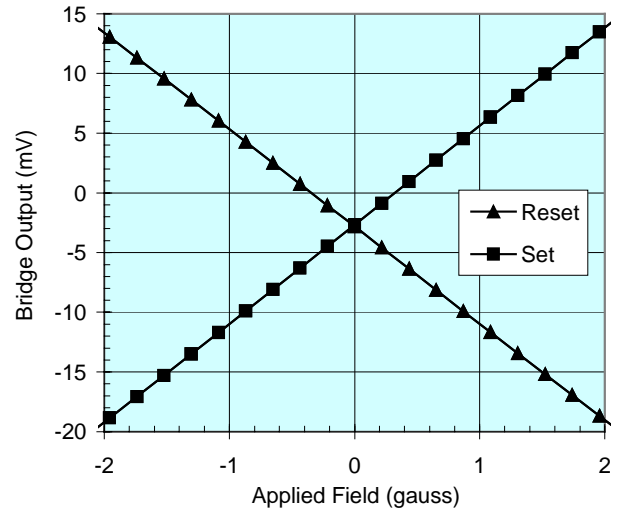


Figure 6—Set and reset output transfer curves.

pulse [7]. The two curves result from an inversion of the gain slope with a common crossover point at the offset voltage. For the sensor in Figure 6, the sensor offset is  $-3$  mV. This from the resistor mismatch during the manufacture process. This offset is not desirable and can be eliminated using the set/reset switching technique described below. Other methods of offset compensation are described in ref. [8].

The sensor offset ( $V_{os}$ ) can be eliminated by using a simple subtraction technique. First apply a set pulse, measure  $H_{applied}$  and store it as  $V_{set}$ —Figure 7. Then apply a reset pulse and store that reading as  $V_{reset}$ . Subtract these two readings to eliminate  $V_{os}$ :

$$V_{set} = S * H_{applied} + V_{os} \quad (5)$$

$$V_{reset} = -S * H_{applied} + V_{os} \quad (6)$$

$$V_{set} - V_{reset} = S * 2 * H_{applied} \quad (7)$$

The sensor sensitivity ( $S$ ) is expressed in mV/gauss. Note that equation (7) has no  $V_{os}$  term. This method also eliminates the amplifier offset as well. Another benefit is that the temperature drift of the sensor offset and the amplifier is eliminated! Now, a low cost amplifier can be used without concern for its offset effects. This is a powerful technique and is easy to implement if the readings are controlled by a low cost microprocessor.

Using this technique to reduce temperature effects can drop the overall variation in magnetic readings to less than  $0.01\%/^{\circ}\text{C}$ . This amounts to less than  $0.29^{\circ}$  effect on the heading accuracy over a  $50^{\circ}\text{C}$  temperature change.

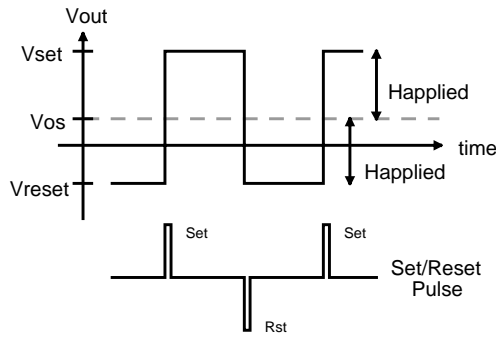


Figure 7—Set and reset effect on sensor output (Vout) show the peak-to-peak level is 2\*Happlied.

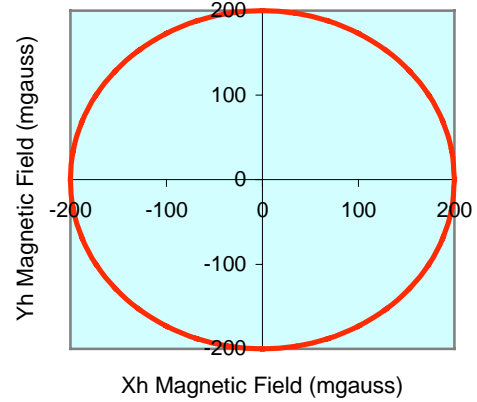
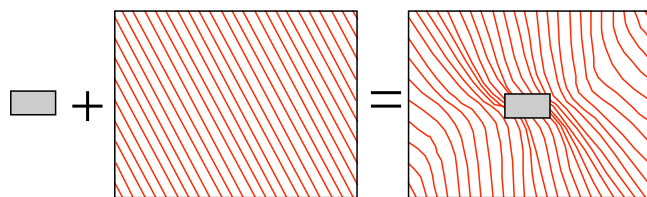


Figure 9—Magnetic sensor outputs (X,Y) rotated horizontally in the earth’s field with no disturbances.

*Nearby Ferrous Materials*—Another consideration for heading accuracy is the effects of nearby ferrous materials on the earth’s magnetic field [9-11]. Since heading is based on the direction of the earth’s horizontal field (Xh, Yh), the magnetic sensor must be able to measure this field without influence from other nearby magnetic sources or disturbances. The amount of disturbance depends on the material content of the platform and connectors as well as ferrous objects moving near the compass.

When a ferrous object is placed in a uniform magnetic field it will create disturbances as shown in Figure 8. This object could be a steel bolt or bracket near the compass or an iron door latch close to the compass. The net result is a characteristic distortion, or anomaly, to the earth’s magnetic field that is unique to the shape of the object.

Before looking at the effects of nearby magnetic disturbances, it is beneficial to observe an ideal output curve with no disturbances. When a two-axis (X,Y) magnetic sensor is rotated in the horizontal plane, the output plot of Xh vs. Yh will form a circle centered at the (0,0) origin (see Figure 9). If a heading is calculated at each point on the circle, the result will be a linear sweep from 0° to 360°.



Ferrous Object + Uniform Magnetic Field = Field Disturbance

Figure 8—Ferrous object disturbance in uniform field.

The effect of a magnetic disturbance on the heading will be to distort the circle shown in Figure 9. Magnetic distortions can be categorized as two types—hard iron and soft iron effects. Hard iron distortions arise from permanent magnets and magnetized iron or steel on the compass platform. These distortions will remain constant and in a fixed location relative to the compass for all heading orientations. Hard iron effects add a constant magnitude field component along each axes of the sensor output. This appears as a shift in the origin of the circle equal to the hard iron disturbance in the Xh and Yh axis (see Figure 10). The effect of the hard iron distortion on the heading is a one-cycle error and is shown in Figure 11.

To compensate for hard iron distortion, the offset in the center of the circle must be determined. This is usually done by rotating the compass and platform in a circle and measure enough points on the circle to determine this offset. Once found, the (X,Y) offset can be stored in memory and subtracted from every reading. The net result will be to eliminate the hard iron disturbance from the heading calculation; as if it were not present[1].

The soft iron distortion arises from the interaction of the earth’s magnetic field and any magnetically soft material surrounding the compass. Like the hard iron materials, the soft metals also distort the earth’s magnetic field lines. The difference is the amount of distortion from the soft iron depends on the compass orientation. Soft iron influence on the field values measured by X and Y sensors are depicted in Figure 12. Figure 13 illustrates the compass heading errors associated with this effect—also known as a two cycle error.

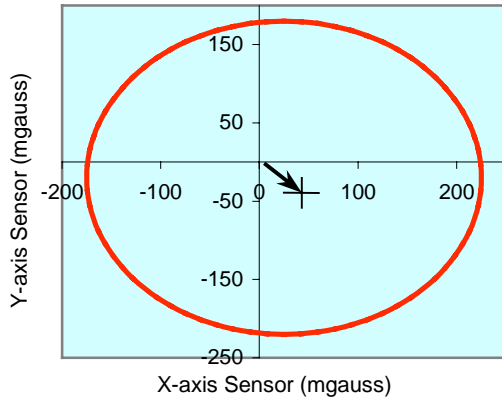


Figure 10— Hard iron offsets when rotated horizontally in the earth's field.

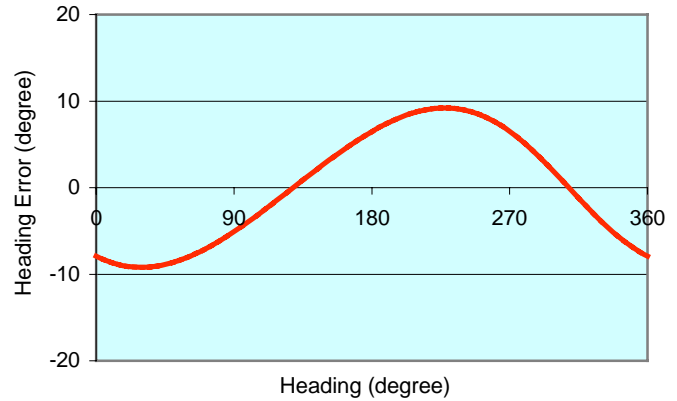


Figure 11— Heading error due to hard iron effects known as single-cycle errors.

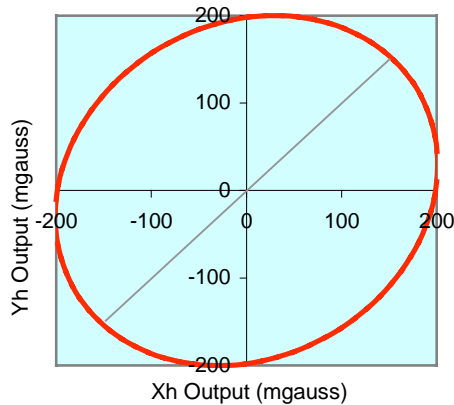


Figure 12— Soft iron distortion when rotated horizontally in the earth's field.

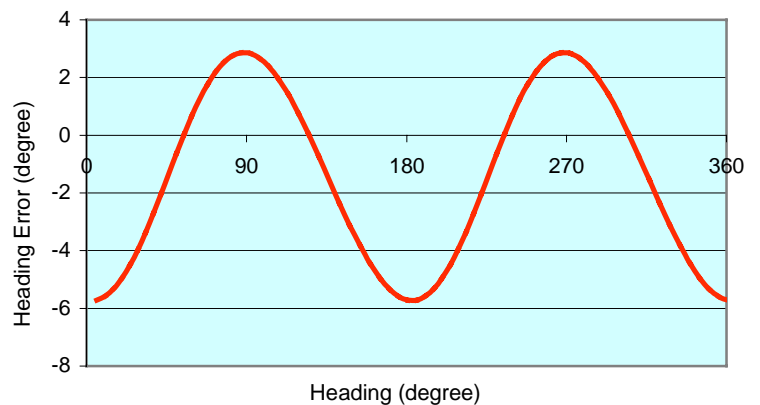


Figure 13— Heading error due to soft iron effects known as two-cycle errors.

Compensating for soft iron effects is a bit more difficult than for hard iron effects. This involves a bit more calculation than a simple subtraction. One way to remove the soft iron effect is to rotate the reading by 45°, scale the major axis to change the ellipse to a circle, then rotate the reading back by 45°. This will result in the desired circular output response shown in Figure 9.

Most ferrous material in vehicles tend to have hard iron characteristics. The best approach is to eliminate any soft iron materials near the compass and deal with the hard iron effects directly. It is also recommended to degauss the platform near the compass prior to any hard/soft iron compensation.

Some compass manufacturers provide calibration methods to compensate for the hard and soft iron effects. Each calibration method is associated with a specified physical movement of the compass platform in order to sample the magnetic space surrounding the compass. The calibration procedure can be as simple as pointing the host in three known directions, or as complicated as moving in a complete circle with pitch and roll, or

pointing the host in 24 orientations including variations in tilt. It is impossible for a marine vessel to perform the 24-point calibration, but easy for a hand-held platform. If the compass is only able to sample the horizontal field components during calibration, then there will be uncompensated heading errors with tilt. Heading error curves can be generated for several known headings to improve heading accuracy [10,11].

Hard and soft iron distortions will vary from location to location within the same platform. The compass has to be mounted permanently to its platform to get a valid calibration. A particular calibration is only valid for that location of the compass. If the compass is reoriented in the same location, then a new calibration is required. A gimballed compass can not satisfy these requirements and hence the advantage of using a strapdown, or solid state, magnetic sensor. It is possible to use a compass without any calibration if the need is only for repeatability and not accuracy.



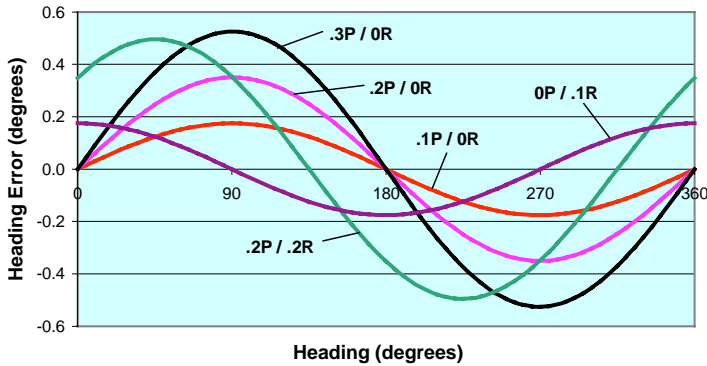


Figure 14—Heading error due to roll and pitch tilt errors (.2P/.2R = .2° error in pitch and roll).

*Compass Tilt Errors*—Heading errors due to the tilt sensor depend somewhat on geographic location. At the equator, tilt errors are less critical since the earth's field is strictly in the horizontal plane. This provides larger Xh and Yh readings and little Z component correction [ref. Equation (2)]. Near the magnetic poles, tilt errors are extremely important—since there is less Xh,Yh field and more Z component. Tilt errors are also dependent on the heading [ref. Figure 4].

Tilt sensors also have offset, gain errors, and temperature effects that need to be accounted for. These will not be compensated for during hard/soft iron calibration, as in the case for the magnetic sensors. The offset error can be zeroed out after installation and will include any platform leveling error. Also, temperature drifts, linearity, repeatability, hysteresis and cross-axis effects are important. The tilt sensor usually contributes the largest percentage of error to the heading calculation.

For a one degree compass, a tilt sensor with 0.1° resolution is desired. The total error introduced by the tilt sensor should be less than 0.5°. The curve in Figure 14 shows the effect on heading for various tilt sensor errors. In this Figure, a pitch error of 0.3° and no roll error can contribute a 0.5° error alone.

*Variation of the Earth's Field*—The final consideration for heading accuracy is the variation, or declination, angle. It is well known that the earth's magnetic poles and its axis of rotation are not at the same geographical location. They are about 11.5° rotation from each other. This creates a difference between the true north, or grid north, and the magnetic north, or direction a magnetic compass will point. Simply it is the angular difference between the magnetic and true north expressed as an Easterly or Westerly variation. This difference is defined as the variation angle and is dependent on the compass

location—sometimes being as large as 25°. To account for the variation simply add, if Westerly, or subtract, if Easterly, the variation angle from the corrected heading computation.

The variation angles have been mapped over the entire globe. For a given location the variation angle can be found by using a geomagnetic declination map or a GPS (Global Positioning System) reading and an IGRF model. The International Geomagnetic Reference Field (IGRF) is a series of mathematical models describing the earth's field and its time variation [12-14]. After heading is determined, the variation correction can be applied to find true north according to the geographic region of operation.

## COMPASS INSTALLATION

The performance of a compass will greatly depend on its installation location. A compass depends on the earth's magnetic field to provide heading. Any distortions of this magnetic field by other sources should be compensated for in order to determine an accurate heading. Sources of magnetic fields include permanent magnets, motors, electric currents—either dc or ac, and magnetic metals such as steel or iron. The influence of these sources on compass accuracy can be greatly reduced by placing the compass far from them. Some of the field effects can be compensated by calibration. However, it is not possible to compensate for time varying magnetic fields; for example, disturbances generated by the motion of magnetic metals, or unpredictable electrical current in a nearby wire. Magnetic shielding can be used for large field disturbances from motors or speakers. The best way to reduce disturbances is distance. Also, never enclose the compass in a magnetically shielded metallic housing.

## ACCELERATION EFFECTS

Any acceleration of the compass will effect the tilt or accelerometer outputs and will result in heading errors. An aircraft making a turn will cause the tilt sensors to experience the centripetal force in addition to gravity and the compass heading will be in error. However, for most applications the acceleration is small, or is in effect for a short duration, making a magnetic compass a useful navigation tool. Inertial reference systems would be the solution for applications that can not tolerate these heading errors. These systems would weigh, cost, and consume power at least 10 times more than those of a strapdown magnetic compass.

## CONCLUSION

A low cost compass has been discussed here having a one degree accuracy requirement. At the heart of the compass is a three-axis MR magnetic sensor and a two-axis electrolytic tilt sensor. Other circuits include a 12 to 14 bit A/D converter, signal conditioning electronics and a microprocessor. The error budget for heading accuracy breaks down as:

Magnetic sensor error	0.14°
Temperature effects	0.29°
Signal conditioning	0.05°
Tilt sensor error	<u>0.50°</u>
Total Error	0.98°

The effects of nearby magnetic distortions can be calibrated out of the compass readings once it is secured to the platform. Caution must be taken in finding a compass location that is not too near varying magnetic disturbances and soft iron materials. Shielding effects from speakers and high current conductors near the compass may be necessary.

Variations in the earth's field from a true north heading can be accounted for if the geographical location of the compass is known. This can be achieved by using a map marked with the deviation angles to find the correct heading offset variation; or use a GPS system and the IGRF reference model to compute the variation angle.

Low cost compasses of the type described here are susceptible to temporary heading errors during accelerations and banked turns. The heading accuracy will be restored once these accelerations diminish. With a strapdown compass there is no accuracy drift to worry about since the heading is based on the true earth's magnetic field. They tend to be very rugged to shock and vibration effects and consume very low power and are small in size.

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### Unit conversion from SI to Gaussian:

79.6 A/m = 1 oersted

1 gauss = 1 oersted (in free air)

1 gauss =  $10^{-4}$  tesla =  $10^5$  gamma

1 nanotesla = 10 microgauss = 1 gamma